E104 Lecture 9 - 20 October 1999

Second order systems - forced response, transfer functions, total response

Announcements

- Midterm on 10/27
- Lab 1 due this Friday
- PS6 (originally due Mon 10/25) can be turned in on Wed at the midterm

Finish Natural response for 2nd order

- s plane
- generality of exponential and sinusoidal responses
- superposition of responses from all roots, but some may persist longer due to longer time constant
- show equivalence of

$$x(t) = C_1 e^{(-\sigma + j\omega)t} + C_2 e^{(-\sigma - j\omega)t} \text{ and } x(t) = Ce^{-\sigma t}\cos(\omega t - \psi)$$

C1 and C2 can be complex, so $C_1 = C_{10}e^{j\phi_1}$ and $C_2 = C_{20}e^{-j\phi_2}$

$x(t) = e^{-\sigma t}[C_{10}e^{j(\omega t+\phi_1)} + C_{20}e^{j(\omega t+\phi_2)}]$

Apply Euler's equation, $e^{j\phi} = \cos\phi + j\sin\phi$

$x(t) = e^{-\sigma t}[C_{10}\cos(\omega t+\phi_1) + jC_{10}\sin(\omega t+\phi_1) + C_{20}\cos(\omega t+\phi_2) - jC_{20}\sin(\omega t+\phi_2)]$

since x(t) is a real physical quantity, it cannot have an imaginary component, so:

$jC_{10}\sin(\omega t+\phi_1) - jC_{20}\sin(\omega t+\phi_2) = 0$

for this to be true for all t, $\phi_1 = \phi_2$ and $C_{10} = C_{20}$

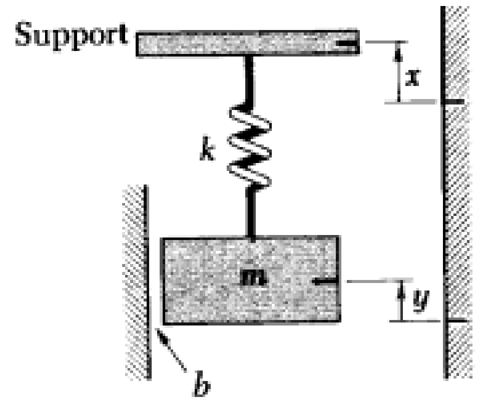
so, rewriting x(t),

$x(t) = e^{-\sigma t}[2C_{10}\cos(\omega t+\phi_1)]$

if we choose $C = 2C_{10}$ and $\psi = -\phi_1$, then we get

$$x(t) = Ce^{-\sigma t}\cos(\omega t - \psi)$$

2nd order system - forced, total response (example 8.9)



Complex numbers - Cartesian ==> mag, angle form

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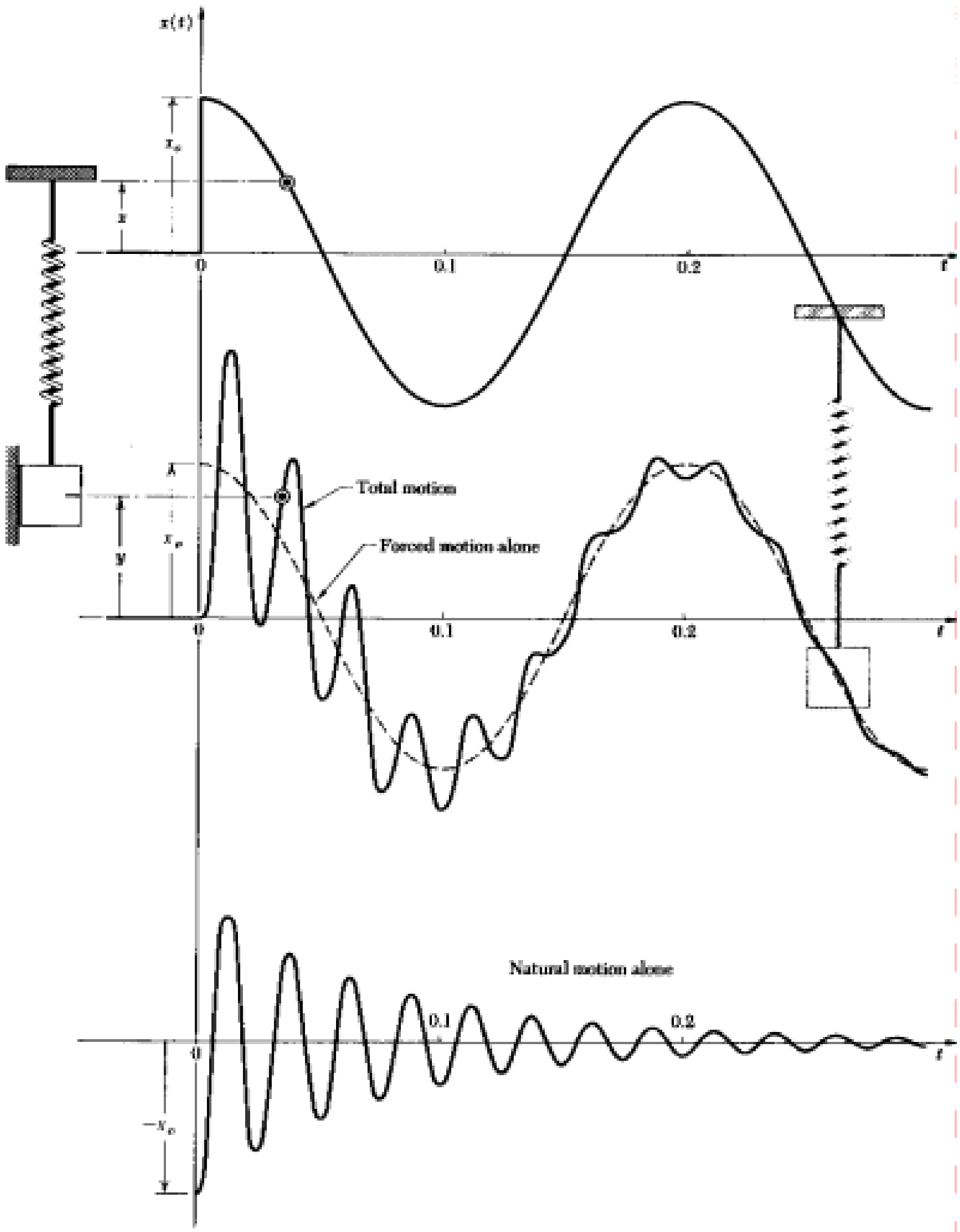
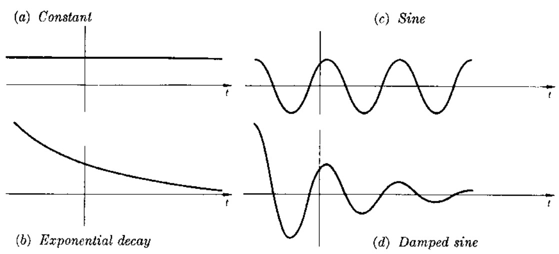


Fig. 8.25 *The total response of a second-order system to an abrupt sinusoidal input*

Forced Response, Transfer functions

The forcing inputs (and therefore the forced responses) can be represented in the form Ae^{st}. Doing this for forced response (as we already do for natural response) will streamline our solution methods.

We will have one "s" for forced response and one "s" for natural response ==> they're different



$$\left.\begin{array}{lll} \text{General} & M = \mathsf{M}e^{st} & \\ \text{Constant} & M = \mathsf{M} & (s = 0) \\ \text{Exponential} & M = \mathsf{M}e^{-\sigma_f t} & (s = -\sigma_f) \\ \text{Sinusoid‡} & M \stackrel{r}{=} \mathsf{M}e^{j\omega_f t} & (s = j\omega_f) \\ \text{Damped sinusoid‡} & M \stackrel{r}{=} \mathsf{M}e^{(-\sigma_f + j\omega_f)t} & (s = -\sigma_f + j\omega_f) \end{array}\right\} \quad (8.21)$$

Forced Response always has the same "form" as the forcing input.

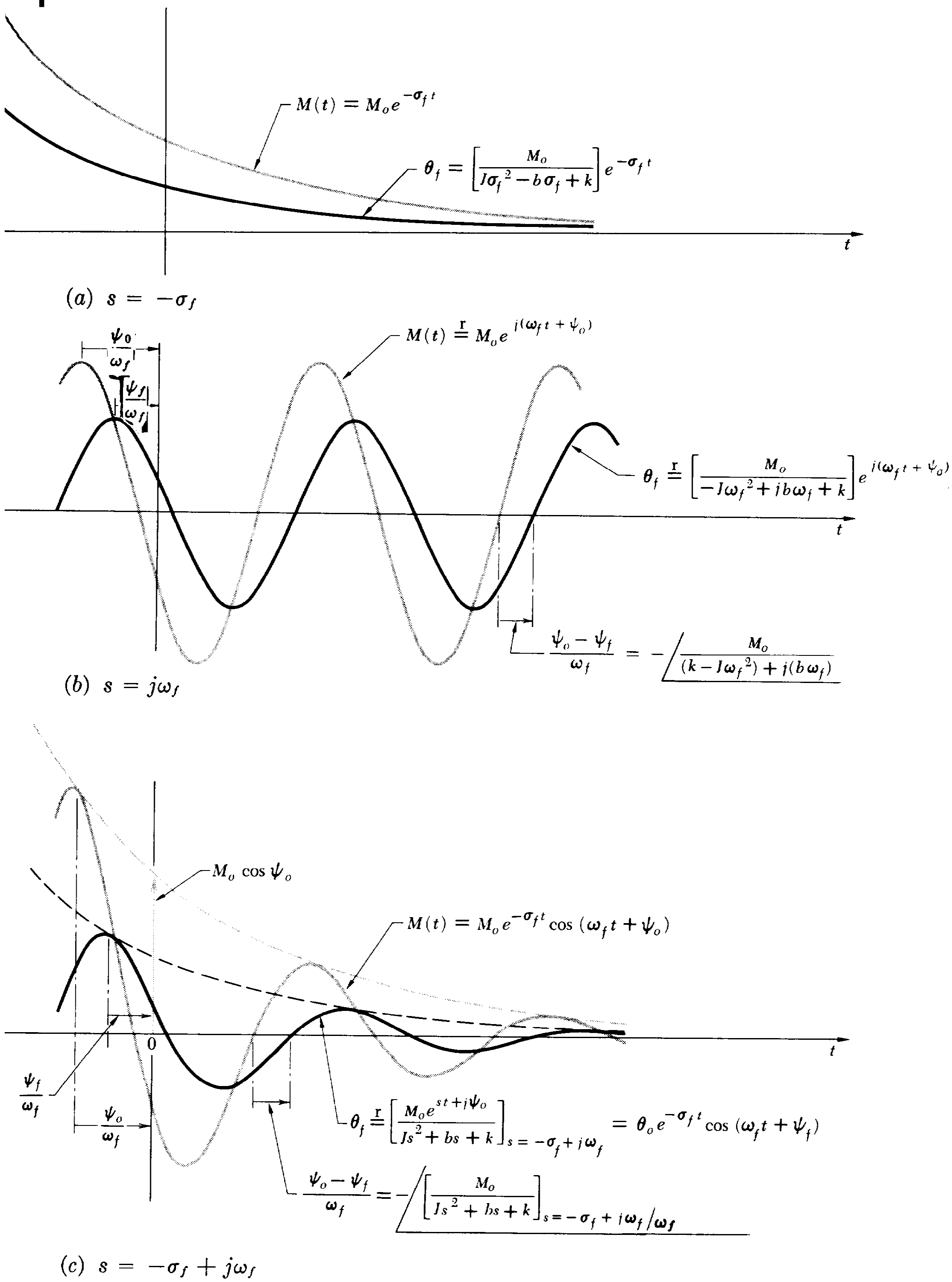


Fig. 8.14 *Forced response alone of a second-order system, when the input has the general form* $M(t) = Me^{st}$

Recap total response for 1st and 2nd order systems

- **Total response** is sum of natural and forced responses
- **Natural response** has form $\mathbf{e^{st}}$, where the values of s (characteristics, roots of CE, eigenvalues) are functions of the system's physical parameters (m,b,k, etc.). s plane
- **Forced response** has form $\mathbf{e^{st}}$ (different s from natural response) where s is the same as in the forcing function. Multiple forced responses can be superimposed.
- **Transfer function** useful for representing/calculating cause==>effect between input and output
- **Complex math** was introduced to facilitate solution - exponential and sinusoidal effects can be handled simultaneously.
- **Time response** (from experiment) can be used to find system characteristics and physical parameters (later, we'll use frequency response to do this as well)